

## SINGLE PARTICLE ELECTROPRODUCTION AT LARGE $Q^2$ AND THE ROLE OF WEE PARTONS \*

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A parton model is presented for single particle electroproductions with large electronic momentum-transfers. Two different asymptotic limits are discussed and related to two kinds of parton-mechanisms. Several testable predictions are made.

The parton model is now widely known for its many predictions in inclusive deep inelastic lepton-hadron processes. However, it can also be applied to certain exclusive reactions where the very soft or wee \*\* [1] partons play the dominant role. These wees carry a fraction  $\mu/\sqrt{Q^2}$  \*\*\* of the relevant hadron's momentum in the infinite momentum frame. Drell and Yan [2] encountered such wee partons when they obtained a connection between large  $Q^2$  elastic ep scattering and near-threshold deep inelastic ep scattering.

We have applied the approach of ref. [2] in the field-theoretic parton model of Drell et al. [3] to the reaction  $ep \rightarrow e^+AB$  (A, B hadrons) in the limit of large  $Q^2$  \*\*\*. We obtain strong dynamical restrictions on certain kinematic variables in appropriate asymptotic limits and suggest definite asymptotic behaviors on the part of a structure-function of interest in those limits. The timeliness of this investigation is underscored by current experiments in progress at the Cornell electron-synchrotron on processes such as  $ep \rightarrow \pi^0 p$ ,  $\rho^0 p$ ,  $\pi^+ n$ ,  $\rho^+ n$ , etc. at large  $Q^2$ . These can test some of the theoretical results obtained with the parton approach. In this letter, we briefly report the results of our work and bring out their physical content. The details are contained in a lengthier publication [4] which follows.

Relevant notations for our process are introduced in fig.1. The three independent scalar variables can be taken to be  $Q^2$ ,  $\nu$  and  $\kappa_A$  where A is the detected particle. We shall assume, for convenience, that this particle is being detected close to the direction of the virtual photon in the laboratory, although the extension of our kinematic discussion to a different geometry of detection is quite straightforward.

If the azimuthal angle, made by  $p_A$  in the plane perpendicular to the direction of the virtual photon, integrated away, we can write differential cross sections as follows [5]:

$$\frac{d^3\sigma}{dQ^2 d\nu d\kappa_A} = \frac{4\pi\alpha^2\epsilon'}{Q^4\epsilon} = \left[ \mathcal{W}_2(Q^2, \nu, \kappa_A) \cos^2 \frac{1}{2} \theta_e + 2\mathcal{W}_1(Q^2, \nu, \kappa_A) \sin^2 \frac{1}{2} \theta_e \right] \quad (1)$$

The structure-functions  $\mathcal{W}_{1,2}$  in eq. (1) are defined by [5]:

$$\begin{aligned} \frac{p_0}{M_p} \sum_{\text{spins}} \int d^3 p_A \int d^3 p_B \delta\left(\kappa_A - \frac{p \cdot p_A}{M_p}\right) (2\pi)^6 \delta^{(4)}(p + q - p_A - p_B) \langle p | J_\mu | p_A, p_B \rangle \langle p_A, p_B | J_\nu | p \rangle \\ = \left( -g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} \right) \mathcal{W}_1(Q^2, \nu, \kappa_A) + \frac{1}{M_p^2} \left( p_\mu - \frac{p \cdot q}{q^2} q_\mu \right) \left( p_\nu - \frac{p \cdot q}{q^2} q_\nu \right) \mathcal{W}_2(Q^2, \nu, \kappa_A) \quad (2) \end{aligned}$$

In eq. (2),  $J_\mu$  stands for the hadronic electromagnet current and  $\sum_{\text{spins}}$  implies that the initial spins are averaged and the final spins summed.  $\mathcal{W}_2$  is the structure function about which we shall be able to make any statement.

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\*\* For a lucid discussion of the contrasting dynamical roles of wee and nonwee partons see ref. [1].

\*\*\* Here  $-Q^2$  is the leptonic momentum-transfer squared and  $\mu$  is a mass of the order of the transverse momentum cutoff seen in hadronic reactions. We shall sometimes use the notation  $m$  also for the latter.

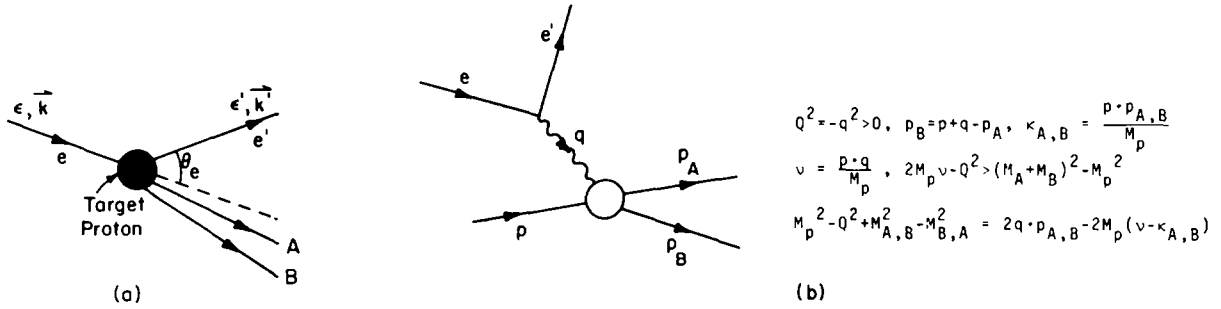


Fig.1. Notations: (a) Laboratory frame, (b) covariantly.

The two asymptotic limits of present interest are the following: (1) Fixed  $\omega$  limit - Here  $\nu \rightarrow \infty$ ,  $Q^2 \rightarrow \infty$  with  $2M_p \nu / Q^2 \equiv \omega$  fixed ( $>1$ );  $\kappa_A$  is a free variable in this case with the restriction  $\kappa_A \leq \nu$ . (2) Fixed  $\tau_A$  limit - Here  $\nu \rightarrow \infty$ ,  $\kappa_A \rightarrow \infty$  with  $\kappa_A / \nu \equiv \tau_A$  fixed ( $<1$ ); now  $Q^2$  is a free variable that can have any large (for the parton-model to be valid) value with  $Q^2 \leq 2M_p \nu$ . The parton-analysis is performed in the infinite momentum frame ( $P \rightarrow \infty$ ) of ref. [2] where

$$p^\mu = (P + M_p^2/2P, 0, 0, P), \quad q^\mu = (M_p \nu / P, \mathbf{q}_\perp, 0) \quad \text{and} \quad q_\perp^2 = Q^2 + O(1/P^2). \quad (3)$$

As explained in ref. [3], we can first take  $P \rightarrow \infty$  with  $q_\perp$  held fixed and then let  $q_\perp$  go to infinity. The four-momenta of the final particles in this frame can be taken to be:

$$p_J^\mu = (X_J P + (M_J^2 + k_{J\perp}^2)/2X_J P, \mathbf{k}_{J\perp}, X_J P) \quad , \quad (4)$$

where  $J$  can be either A or B and  $X_J$  and  $k_{J\perp}$  are unknown quantities. We now have:

$$\kappa_J \equiv p \cdot p_J / M_p = \frac{1}{2} X_J M_p + (M_J^2 + \kappa_{J\perp}^2) / 2X_J M_p \quad (5)$$

On the other hand, momentum- and energy-conservation imply that

$$X_A + X_B = 1 \quad , \quad (6a)$$

$$k_{A\perp} + k_{B\perp} = \mathbf{q}_\perp \quad , \quad (6b)$$

$$M_p^2 + 2M_p \nu = (M_A^2 + k_{A\perp}^2) / X_A + (M_B^2 + k_{B\perp}^2) / X_B \quad . \quad (6c)$$

Eqs. (5) and (6) are sufficient to determine  $X_J$  and  $k_{J\perp}$  for our purposes.

The kinematic restrictions given by the parton model are consequences of the behavior of the wees partons. The presence of wees in exclusive reactions follows from the basic requirement that the transverse momenta involved in any hadronic vertex with partons remain finite (i.e. under a cutoff) even as  $q_\perp \rightarrow \infty$ . The point under consideration can be illustrated by considering elastic ep scattering à la Drell-Yan [2]. In the  $P \rightarrow \infty$  frame let the  $i^{\text{th}}$  parton have the four-momentum:

$$p_i^\mu = [\eta_i P + (\mu_i^2 + k_{i\perp}^2) / 2\eta_i P, \mathbf{k}_{i\perp}, \eta_i P] \quad , \quad (7)$$

where  $0 < \eta_i < 1$ ,  $|k_{i\perp}| < \text{the } k_\perp \text{ - cutoff}$ ,  $\sum_i \eta_i = 1$ ,  $\sum_i \mathbf{k}_{i\perp} = 0$ . The constituent 'a' which gets electromagnetically scattered (fig. 2a) carrying off a momentum  $\eta_a \mathbf{p} + \mathbf{k}_{a\perp} + \mathbf{q}_\perp$  has to combine with the unscattered bunch of momentum  $(1 - \eta_a) \mathbf{p} - \mathbf{k}_{a\perp}$  in the redressing vertex to give back a final physical proton of momentum  $\mathbf{p} + \mathbf{q}_\perp$ . Relative to the last momentum, the first two can be written as  $\eta_a(\mathbf{p} + \mathbf{q}_\perp) + (1 - \eta_a) \mathbf{q}_\perp + \mathbf{k}_{a\perp}$  and  $(1 - \eta_a)(\mathbf{p} + \mathbf{q}_\perp) - (1 - \eta_a) \mathbf{q}_\perp - \mathbf{k}_{a\perp}$  respectively. Hence, with respect to the momentum of the final proton, the transverse momenta at the redressing vertex can be finite (as  $q_\perp \rightarrow \infty$ ) only if  $\eta_a \xrightarrow{q_\perp \rightarrow \infty} 1 - \mu q_\perp^{-1}$ . This means that only 'a' parton is nonwee and the rest are wees.

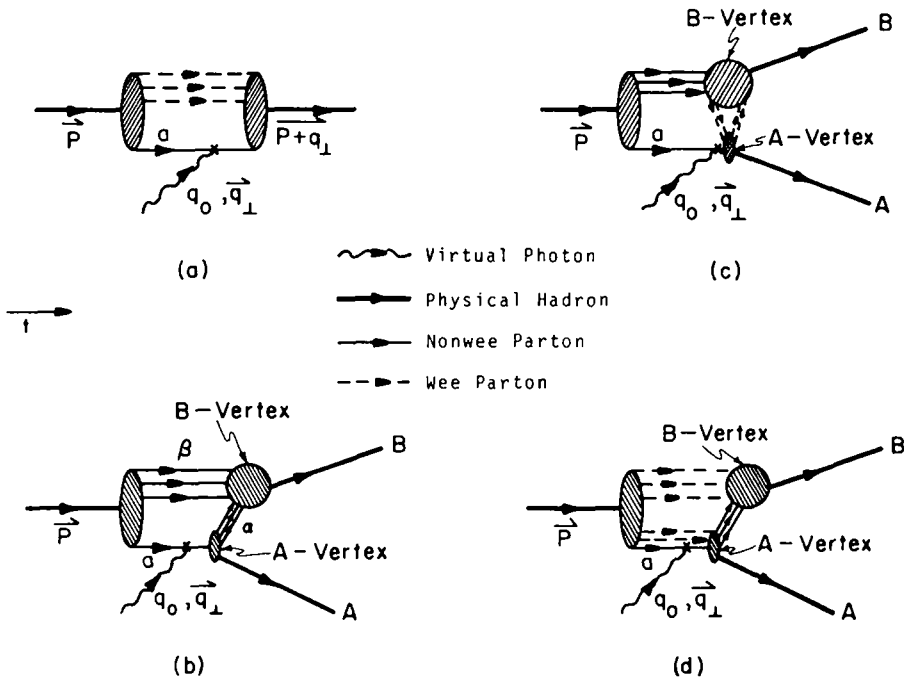


Fig.2. Parton descriptions: (a) large  $Q^2$  elastic ep scattering, (b) Interaction between the electromagnetically scattered bunch in  $ep \rightarrow e'AB$ , (c) Asymptotic single particle electroproduction in the fixed  $\omega$  limit. (d) same but in the fixed  $\tau_A$  limit.

In the reaction  $ep \rightarrow e'AB$ , some interaction is necessary \* between the electromagnetically scattered parton of momentum  $\mathbf{p}_S = \eta_a \mathbf{p} + \mathbf{q}_\perp + \mathbf{k}_{a\perp}$  and the unscattered bunch in order to produce two final hadrons. Consider a particular type of interaction as shown in fig. 2b. The bunch  $\alpha$  of partons, produced at the A-vertex, and the originally unscattered bunch  $\beta$ , produced at the initial undressing vertex, combine at the B-vertex to produce the particle B. Since these are time-ordered diagrams, energy is not conserved at any of these vertices although there is overall energy-conservation. Let  $\chi$  be the fraction of  $\mathbf{p}_S$  imparted to the particle A at the A-vertex, i.e.  $\mathbf{p}_A = \chi \mathbf{p}_S + \mathbf{p}'$  or

$$\mathbf{p}_A = \chi(\eta_a \mathbf{P} + \mathbf{q}_\perp) + \chi \mathbf{k}_{a\perp} + \mathbf{p}' \quad , \quad (8)$$

where  $\mathbf{p}' \cdot \mathbf{p}_S = 0$  and  $\mathbf{p}'$  is finite. The momentum  $(1 - \eta_a) \mathbf{p}_S - \mathbf{p}' = (1 - \chi)(\eta_a \mathbf{p} + \mathbf{q}_\perp) + (1 - \chi) \mathbf{k}_{a\perp} + \mathbf{p}'$ , carried by the bunch  $\alpha$ , and the momentum  $(1 - \eta_a) \mathbf{p} - \mathbf{k}_{a\perp}$ , belonging to the bunch  $\beta$ , give

$$\mathbf{p}_B = (1 - \eta_a \chi) \mathbf{P} + (1 - \chi) \mathbf{q}_\perp + \chi \mathbf{k}_{a\perp} - \mathbf{p}' \quad (9)$$

at the B-vertex. Clearly, the requirement of finite transverse momenta with respect to  $\mathbf{p}_B$  at the B-vertex can be maintained as  $q_\perp \rightarrow \infty$  only if either 1)  $1 - \chi \xrightarrow{q_\perp \rightarrow \infty} \mu q_\perp^{-1}$ , i.e. the bunch  $\alpha$  consists of wee partons only, or if 2)  $1 - \eta_a \xrightarrow{q_\perp \rightarrow \infty} m q_\perp^{-1}$ , i.e. if the bunch  $\beta$  consists solely of wee partons. The first alternative corresponds to wee-exchanges between the A- and B- vertices and in general these wees can go from either vertex to the other \*\*, as shown in fig. 2c. The second possibility corresponds

\* This is because in the model of ref. [3] the absence of any pointlike behavior in elastic ep scattering is achieved by the requirement that a single bare parton cannot make a transition to a physical particle (see the discussion towards the end of the first paper in ref. [3]). Thus we must have at least three final hadrons in order to avoid any interaction between the electromagnetically scattered parton and the unscattered bunch.

\*\* Everything must, of course, go forward with time. This can be insured by considering only the 'good' ( $\mu = 0.3$ ) components of  $J_\mu$ . See the second paper in ref. [3].

to the case when at the initial undressing vertex only the 'a' parton is nonwee and the rest are wees; in the general situation, of course, some of these wees can be absorbed at the A-vertex as shown in fig. 2d. Figs. 2c and 2d illustrate the general description of leading single-particle electroproduction via the two different possible parton mechanisms as  $q_{\perp} \rightarrow \infty$ .

Consider now the situation portrayed in fig. 2c. Let the group of wees moving from the B-vertex to the A-vertex bring in a total momentum of  $(\chi_1 - 1)\mathbf{p}_S + \mathbf{p}'_1$  to the former, whereas those going from the A-vertex to the B-vertex take away a total momentum of  $(1 - \chi_2)\chi_1\mathbf{p}_S + \mathbf{p}'_2$  from the same. Here  $\chi_{1,2} \sim \frac{1 \pm \mu_{1,2}}{q_{\perp}^{-1}}$ ,  $\mu_{1,2} \sim k_{\perp}$  - cutoff,  $\mathbf{p}'_{1,2} \cdot \mathbf{p}_S = 0$  and  $p'_{1,2}$  are finite. Now eqs. (8) and (9) are still valid in this more general situation provided we define  $\chi = \chi_1\chi_2 \sim 1 + (\mu_1 - \mu_2)q_{\perp}^{-1}$  and  $\mathbf{p}' = \mathbf{p}'_1 - \mathbf{p}'_2$ . Comparing eq. (8) and eq. (4) for  $J = A$ , we have  $k_{A\perp} \sim \{1 + (\mu_1 - \mu_2)q_{\perp}^{-1}\} (k_{a\perp} + q_{\perp}) + \text{finite}$  and  $X_A \sim \eta_a [1 + (\mu_2 - \mu_1)q_{\perp}^{-1}]$ , i.e. now

$$\lim_{\substack{q_{\perp} \rightarrow \infty \\ \text{case 1}}} \frac{k_{A\perp} - q_{\perp}}{q_{\perp}} = O\left(\frac{1}{q_{\perp}}\right) \quad (10a)$$

$$\lim_{\substack{q_{\perp} \rightarrow \infty \\ \text{case 1}}} X_A = \eta_a + O\left(\frac{1}{\sqrt{Q^2}}\right) \quad (10b)$$

Eqs. (10), substituted in eq. (6c), imply that

$$\eta_a = \lim_{\substack{q_{\perp} \rightarrow \infty \\ \text{case 1}}} X_A = \frac{Q^2}{2M_p\nu} + O\left(\frac{1}{\sqrt{Q^2}}\right) = \frac{1}{\omega} + O\left(\frac{1}{\sqrt{Q^2}}\right) \quad (11)$$

Since in this case  $\eta_a$  is some finite number in the range  $0 < \eta_a < 1$ , eq. (11) shows that the present situation corresponds to the fixed  $\omega$  limit. Now eqs. (10a) and (11), in conjunction with eq. (5) and the relation  $M_p^2 - Q^2 + M_A^2 - M_B^2 = 2q \cdot p_A - 2M_p(\nu - \kappa_A)$  (vide fig. 1), leads to the following restriction on the free variable  $\kappa_A$  of the fixed  $\omega$  limit:

$$\lim_{\omega} \frac{\kappa_A}{\nu} = \lim_{\omega} \frac{2q \cdot p_A}{-Q^2} = 1 + O\left(\frac{1}{\sqrt{Q^2}}\right) \quad (12)$$

Moreover, if  $\theta_A$  is the laboratory angle between  $\mathbf{p}_A$  and  $\mathbf{q}$ , eq. (12) implies that \*

$$\lim_{\omega} (1 - \cos \theta_A) = O(1/\sqrt{\nu^3}) \quad ,$$

i.e. in the fixed  $\omega$  limit one of the final particles (taken here to be A) emerges predominantly in the laboratory direction of the virtual photon.

The situation illustrated in fig. 2d can be discussed in a similar way. In this case  $\eta_a \sim 1 - m q_{\perp}^{-1}$ . Comparing eq. (8) and eq. (4) for  $J = A$ , we now get

$$\lim_{\substack{q_{\perp} \rightarrow \infty \\ \text{case 2}}} k_{A\perp} = \chi q_{\perp} + \text{finite} \quad , \quad (13a)$$

$$\lim_{\substack{q_{\perp} \rightarrow \infty \\ \text{case 2}}} X_A = \chi + O\left(\frac{1}{\sqrt{Q^2}}\right) \quad (13b)$$

\* In the fixed  $\omega$  limit, the result  $-2q \cdot p_A Q^{-2} = 1 + O(1/\sqrt{Q^2})$  means that in the laboratory, as  $Q^2 \rightarrow \infty$ , we have  $-2Q^{-2} \{ \nu \kappa_A - \sqrt{\nu^2 + Q^2} \sqrt{\kappa_A^2 - M_A^2} \cos \theta_A \} = 1 + O(1/\sqrt{Q^2})$ . Since now  $\kappa_A/\nu = 1 + O(1/\sqrt{\nu})$ , after expanding the square root, we can write  $(1 + M_p^{-1} \omega \kappa_A) (1 - \cos \theta_A) = O(1/\sqrt{\nu})$  which immediately leads to the desired result. In the fixed  $\tau_A$  limit, the result  $-M_p^{-1} \nu^{-1} q \cdot p_A = \tau_A + O(1/\sqrt{\nu})$  implies that in the lab we have  $-M_p^{-1} \nu^{-1} \{ \nu \kappa_A - \sqrt{\nu^2 + Q^2} \sqrt{\kappa_A^2 - M_A^2} \cos \theta_A \} = \tau_A + O(1/\sqrt{\nu})$ . Since now  $Q^2/2M_p\nu$  is  $1 + O(1/\sqrt{\nu})$ , binomial expansions of the square-roots give  $(1 - \cos \theta_A) (1 + \nu M_p^{-1}) = O(1/\sqrt{\nu})$  which again leads to the result claimed.

Substituted in eq. (5), eqs. (13) yield the result

$$\lim_{\substack{q_{\perp} \rightarrow \infty \\ \text{case 2}}} \frac{\chi_A}{\nu} = \frac{\chi Q^2}{2M_p \nu} + O\left(\frac{1}{\sqrt{\nu}}\right) \quad (14)$$

On the other hand, eq. (6c) and eqs. (13) lead to the relation

$$\lim_{\substack{q_{\perp} \rightarrow \infty \\ \text{case 2}}} \frac{Q^2}{2M_p \nu} = 1 + O\left(\frac{1}{\sqrt{Q^2}}\right) \quad (15)$$

which, combined with eq. (14), implies that

$$\lim_{\substack{q_{\perp} \rightarrow \infty \\ \text{case 2}}} \frac{\chi_A}{\nu} = \chi + O\left(\frac{1}{\sqrt{Q^2}}\right) \quad (16)$$

Since  $\chi$  is now a finite number in the range  $0 < \chi < 1$ , this case corresponds to the fixed  $\tau_A$  limit with  $\chi = \tau_A$ . Eq. (15) is the restriction on the free variable  $Q^2$  in this limit. In analogy with eq. (12), we here have

$$\lim_{\tau_A} q \cdot p_A / (-M_p \nu) = \tau_A + O(1/\sqrt{\nu})$$

which implies that once again \*

$$\lim_{\tau_A} (1 - \cos \theta_A) = O(1/\sqrt{\nu^3})$$

Moreover, eqs. (13) now become:

$$\lim_{\tau_A} X_A = \tau_A + O(1/\sqrt{Q^2}) \quad (17a)$$

$$\lim_{\tau_A} k_{A\perp} = \tau_A q_{\perp} + \text{finite} \quad (17b)$$

The results on the asymptotic behavior of  $\mathcal{W}_2$  in the fixed  $\omega$  and  $\tau_A$  limits can be attained once we understand the role of the wee partons in governing these limits and also in controlling the large  $Q^2$  behavior of the electromagnetic form-factor of any hadron. Take the fixed  $\omega$  limit first. In fig. 2c, which describes the situation in this limit, the wees are exchanged between the A- and B-vertices. Consider now the figures corresponding to fig. 2a for the form-factors  $F_{A, B}$  of the particles A, B (instead of the proton) and compare these with fig. 2c. Suppose we argue - as Drell and Yan did in ref. [2] - that the leading  $q_{\perp}$ -dependence comes solely from the wee-partons and nothing else. Then the contribution to the leading  $q_{\perp}$ -behavior in fig. 2c from a certain configuration of  $l$  wee partons should be related to the contributions to the form-factors of A and B from the same wee partons; the non-wee partons in fig. 2c would simply affect the dependence on the scaling variable  $\omega$ . Since the form-factor of A(B) involves the square of the vertex-function association with the A-(B-) vertex while these occur once each in fig. 2c, we expect the square of the contribution from wee-partons to the single particle electroproduction amplitude in the fixed  $\omega$  limit to be proportional to the product of the contributions to the form-factors from the same wees. Furthermore, if this relation is true for each wee configuration, we can infer that it should be generally valid. The argument is described in detail in ref. [4] leading to the exact result which is:

$$\lim_{\omega} \sum_{\text{spins}} (2\pi)^q \langle p | J^{0,3} | p_A, p_B \rangle \langle p_B, p_A | J^{0,3} | p \rangle = \frac{F_A(Q^2) F_B(Q^2)}{P} u(\omega)$$

\* For a spin-half baryon  $F_1$  is meant and for a neutral particle the electromagnetic form-factor of its charged  $l$ -spin partner.

where  $u(\omega)$  is an unknown function of  $\omega$ . If the hadronic form-factors are all taken \* to fall off with a universal power of  $Q^{-2}$  at large  $Q^2$ , we can drop the subscripts in  $F_{A,B}$  and write the above equation as:

$$\lim_{\omega} \sum_{\text{spins}} (2\tau)^q \langle p | J^{0,3} | p_A, p_B \rangle \langle p_B, p_A | J^{0,3} | p \rangle = \frac{[F(Q^2)]^2}{P} u(\omega) \quad (18)$$

Similarly, for the fixed  $\tau_A$  limit, our result in [4]:

$$\lim_{\tau_A} \sum_{\text{spins}} (2\pi)^q \langle p | J^{0,3} | p_A, p_B \rangle \langle p_B, p_A | J^{0,3} | p \rangle = \frac{[F(Q^2)]^2}{P} v(\tau_A) \quad (19)$$

where  $v(\tau_A)$  is an unknown function of  $\tau_A$ .

The conversion of eqs. (18) and (19) into statements on the behavior of  $\mathcal{W}_2$  can be made by taking  $\mu, \nu = 0, 3$  in eq. (2) and letting  $P$  go to  $\infty$ . Using eqs. (4), (5) and (6), we then have

$$\begin{aligned} \mathcal{W}_2 = & M_p \int_0^1 dX_A \int \pi d^2 k_{A\perp}^2 \delta\left(X_A - \frac{X_A M_p}{2} - \frac{M_A^2 + k_{A\perp}^2}{2 X_A M_p}\right) \\ & \times \delta\left[M_p^2 + 2M_p \nu - \frac{M_A^2 + k_{A\perp}^2}{X_A} - \frac{M_B^2 + (q_{\perp} - k_{A\perp})^2}{1 - X_A}\right] 2^P (2\pi)^6 \sum_{\text{spins}} \langle p | J^{0,3} | p_A, p_B \rangle \langle p_B, p_A | J^{0,3} | p \rangle \end{aligned}$$

For the fixed  $\omega$  limit, we can use eqs. (10a) and (18) in the above equation and obtain:

$$\begin{aligned} \lim_{\omega} \mathcal{W}_2 = & M_p \int_0^1 dX_A \delta\left(X_A - \frac{Q^2}{2M_p \nu}\right) 2 X_A^2 \frac{P}{Q^2} \int \pi d^2 k_{A\perp}^2 \delta(2 X_A X_A M_p - X_A^2 M_p^2 - M_A^2 - k_{A\perp}^2) \\ & \times 2 X_A M_p \left(\frac{1}{2\pi}\right)^3 \frac{1}{P} [F(Q^2)]^2 u(\omega) \quad , \end{aligned}$$

$$\text{or. } \lim_{\omega} \nu \mathcal{W}_2 = [F(Q^2)]^2 U(\omega) \quad (20)$$

where  $U(\omega) = [u(\omega) M_p / (2\pi \omega)^2]$  is an unknown function of  $\omega$ . Similarly, in the fixed  $\tau_A$  limit using eqs. (17) and (19), we obtain:

$$\begin{aligned} \lim_{\tau_A} \mathcal{W}_2 = & M_p \int \pi d^2 k_{A\perp}^2 \delta(k_{A\perp}^2 - 2\tau_A M_p X_A) 2 M_p \tau_A 2^P \int_0^1 dX_A \delta(\tau_A - X_A) \\ & \times \left| \frac{1}{M_A^2/\tau_A^2 - M_B^2/(1-\tau_A)^2} \right| \left(\frac{1}{2\pi}\right)^3 \frac{1}{P} [F(Q^2)]^2 v(\tau_A) \quad , \end{aligned}$$

$$\text{or. } \lim_{\tau_A} \mathcal{W}_2 = [F(Q^2)]^2 V(\tau_A) \quad (21)$$

where  $V(\tau_A) = M_p^2 \tau_A v(\tau_A) \left| M_A^2/\tau_A^2 - M_B^2/(1-\tau_A)^2 \right|^{-1} / 2\pi^2$  is an unknown function of  $\tau_A$ .

In conclusion, let us remark that we have two kinds of parton-model results for single-particle electroproduction in the fixed  $\omega$  and fixed  $\tau_A$  limits. Eqs. (12) and (15) are kinematic restrictions on the free variable in each limit that follow in a straightforward manner from the fundamental ingredients of the parton picture. On the other hand, eqs. (20) and (21) depend on certain additional theoretical assumptions which seem plausible in such a picture. (For a detailed discussion of these assumptions the

\* If large  $Q^2$  elastic electron scattering of a hadron is controlled by the wee partons, one is inclined to expect the asymptotic form-factors of all hadrons to fall off with the same power of  $Q^{-2}$ . This is the point of view that we shall adopt. A more detailed discussion will be found in ref. [4].

reader is referred to our forthcoming lengthier paper [4].) However, the basic qualitative feature of eqs. (20) and (21), namely that  $\nu\mathcal{W}_2$  or  $\mathcal{W}_2$  (depending on the limit) should tend to zero as some power of  $Q^{-2}$ , is inevitable in any self-consistent parton treatment of the problem. This contrasts with the prediction of nontrivial scaling for  $\nu\mathcal{W}_2$  in the fixed  $\omega$  limit for the specific process  $ep \rightarrow e\nu\rho^+$  given by Lee [6] in a model based on the current-field identity. Another result to compare is the prediction of Frishman et al. [7] (from light-cone dominance considerations) that in the fixed  $\omega$  limit  $\nu\mathcal{W}_2$  should behave as  $*(Q^2)^{-\alpha}F_2(\omega, \tau)$  where  $\tau = -(p+q-p_A)^2$  and  $\alpha$  is related to the dimension of the leading field in the operator product expansion. It should not, however, be long before these results are confronted with experiment at Cornell.

The author thanks D.R. Yennie for a remark which led to this investigation. He is especially indebted to T.-M. Yan for many guiding suggestions.

\* In the scaling region, if  $\theta_A$  is taken  $\approx 0$ ,  $\nu\mathcal{W}_2$  is proportional to  $T_2/\omega$  with  $T_2$  as defined in ref. [6]. It should be noted that our results are stronger than those of Frishman et al. since we have the explicit form factors  $F_{A,B}(Q^2)$  and specify that  $\kappa_A \approx \nu$  in the fixed  $\omega$  limit. Moreover, light-cone dominance for exclusive processes - as used in ref. [6] - involves the implicit ad hoc assumption of the insensitivity of the amplitude to large variations in the masses of the final hadrons (see the last paragraph of the second paper quoted in ref. [6]).

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